

## Logarithms

### 1. Simplify

(a)  $2\log_6 3 + \log_6 4$

(b)  $3\log_{10} 2 + \log_{10} 5 - \log_{10} 4$

(c)  $\frac{1}{2}\log_2 144 + \log_2 \frac{1}{3}$

### 2. Solve the equations for $x > 0$

(a)  $3\log_a x + \log_a 2 = \log_a 54$

(b)  $\log_2 x + \log_2 (x - 2) = 3$

(c)  $\log_4 (2x - 1) + \log_4 (x + 1) = \frac{1}{2}$

(d)  $\log_3 6x - \log_3 (x - 2) = 2$

(e)  $\frac{1}{2}\log_x 64 + 2\log_x 2 = 5$

(f)  $2\log_a 10 - \log_a 4 = 2$

### 3. Given $\frac{1}{2}\log_a y = \log_a (x - 3) + 2\log_a 2$ , show that $y = 16(x - 3)^2$ .

### 4. Given $\log_x b = 2\log_x a + 4$ , show that $a = \frac{\sqrt{b}}{x^2}$ .

### 5. (a) Show that $(x - 1)$ is a factor of $x^3 + 8x^2 + 11x - 20$ and hence factorise $x^3 + 8x^2 + 11x - 20$ fully.

(b) Solve  $\log_2 (x + 3) + \log_2 (x^2 + 5x - 4) = 3$ .

### 6. Functions $f$ , $g$ and $h$ are defined on suitable domains by

$$f(x) = x^2 - x + 10 \quad g(x) = 5 - x \quad \text{and} \quad h(x) = \log_2 x$$

(a) Find expressions for  $h(f(x))$  and  $h(g(x))$ .

(b) Hence solve  $h(f(x)) - h(g(x)) = 3$ .

### 7. A curve has equation $y = \log_2 (x + 4) - 1$ .

Find the coordinates of the points where this curve crosses the  $x$  and  $y$  axes.

8. A curve has equation  $y = 3\log_3 (x + 27) - 6$ .

Find the coordinates of the points where this curve crosses the x and y axes.

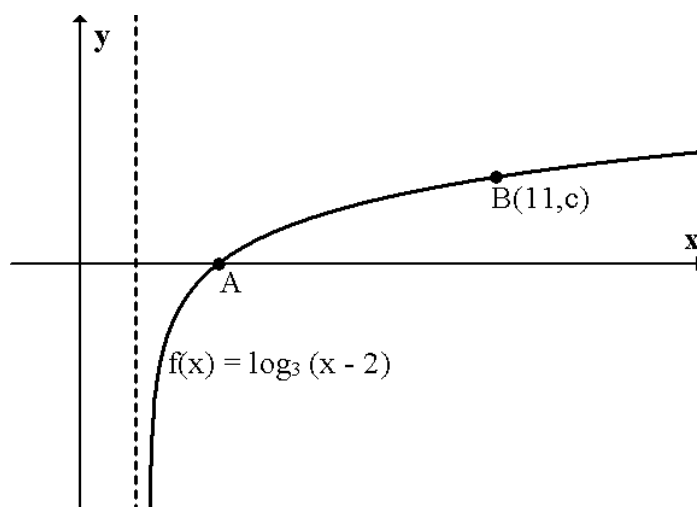
9. The diagram shows the graph of  $f(x) = \log_3 (x - 2)$ .

(a) Find the coordinates of A.

(b) Find the value of c.

(c) Sketch the graph of  $y = \log_3 \frac{1}{x}$

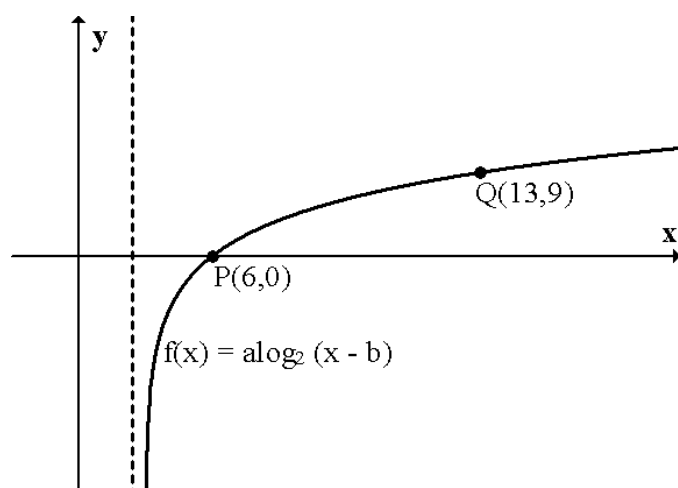
(d) Sketch the graph of  $y = \log_3 (x - 2)^2$



10. The diagram shows the graph of  $f(x) = a\log_2 (x - b)$ .

(a) Find the values of a and b.

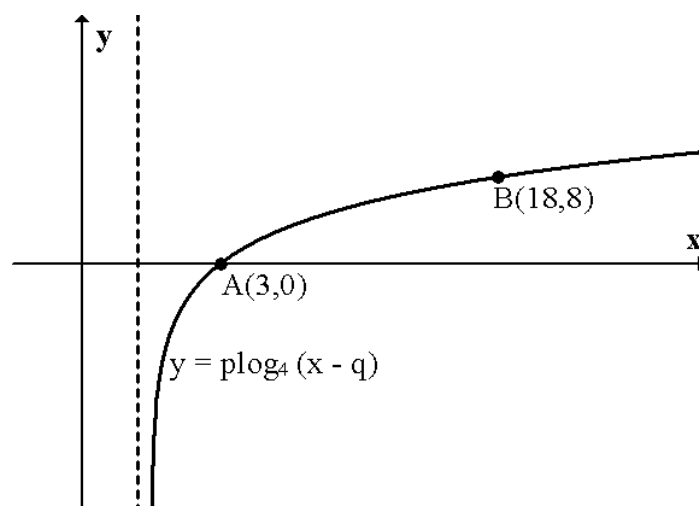
(b) Write down the domain of  $f(x)$ .



11. The diagram shows the graph of  $y = p\log_4 (x - q)$ .

(a) Find the values of p and q.

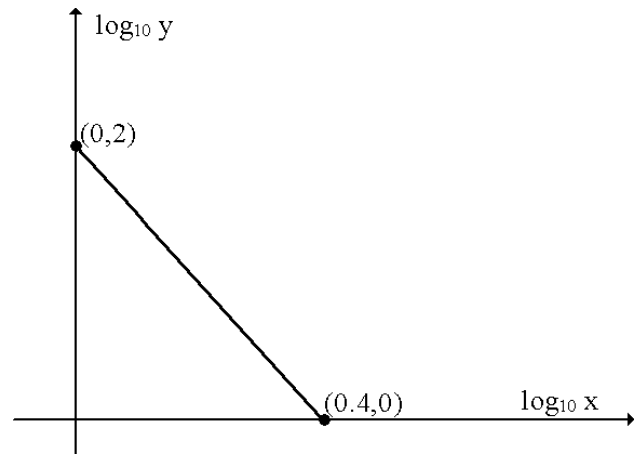
(b) Write down the domain of y.



12. A radioactive material has mass  $m$ , at time  $t$  years, given by  $m = m_0 e^{-0.02t}$ , where  $m_0$  is the original mass.
- If the original mass is 500g, find the mass after 25 years.
  - Find the percentage of the material left after 8 years.
13. For a radioactive substance  $A = A_0 e^{-kt}$ , where  $A_0$  is the original mass and  $t$  is the time in minutes. In 5 minutes, 40g of this substance is reduced to 32g.
- Find  $k$  to 2 significant figures.
  - Find the half life of this substance.
14. The value,  $V$  (£million), of an aeroplane is given by the formula  $V = 3.5e^{-0.095t}$  where  $t$  is the number of years after the aeroplane is put into service..
- Calculate the value of the aeroplane when it was built.
  - How long, to the nearest year, will it take for the aeroplane to fall to 40% of its original value?
15. The mass,  $M$  grams, of a radioactive isotope after a time of  $t$  years, is given by the formula  $M = M_0 e^{-kt}$  where  $M_0$  is the initial mass of the isotope. In 80 years a mass of 40 grams of the isotope is reduced to 30 grams.
- Find the value of  $k$ .
  - Calculate the half-life of the isotope.
  - Calculate the percentage of the isotope left after 1000 years.

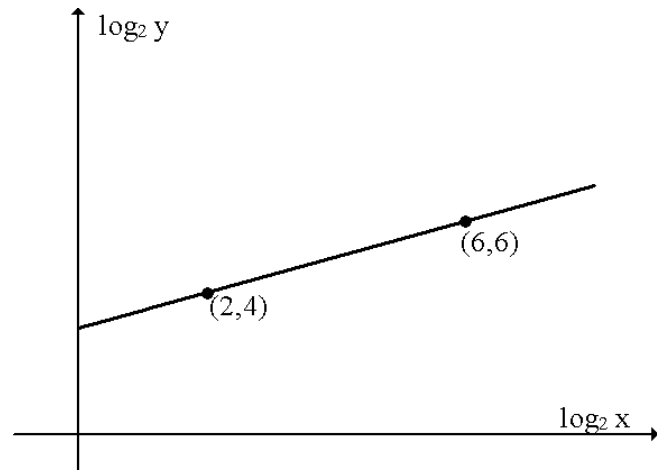
16. The graph opposite shows a relation of the form  $y = kx^n$ .

Find the values of  $k$  and  $n$ .



17. The diagram show a relation of the form  $y = ax^b$ .

Find the values of  $a$  and  $b$ .

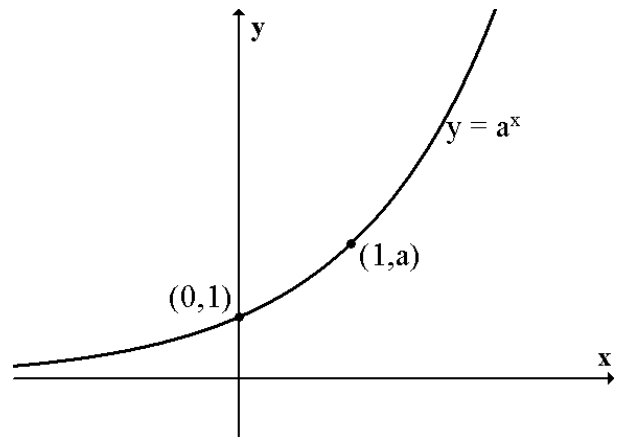


18. (a) The diagram shows the graph of  $y = a^x$ .

(b) Sketch the graphs of

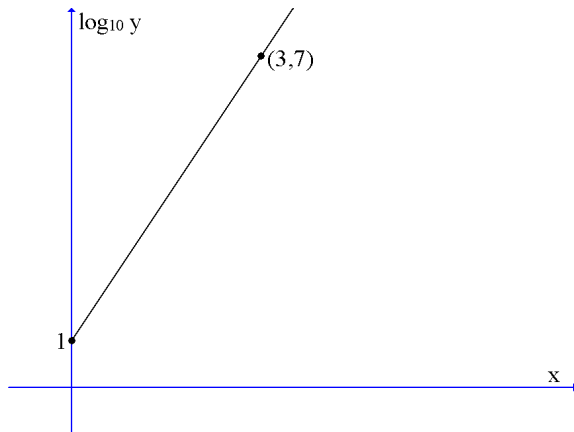
(i)  $y = 2a^x$       (ii)  $y = a^{-2x}$

- (c) Show that the  $x$ -coordinate of the point of intersection of the graphs  $y = 2a^x$  and  $y = a^{-2x}$  is  $x = \frac{1}{3}\log_a \frac{1}{2}$ .

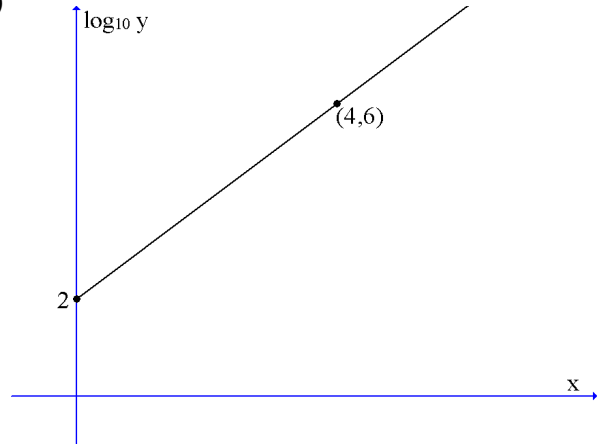


19. Each of the following is of the form  $y = ab^x$ . Find  $a$  and  $b$ .

(a)



(b)



### EXTRAS

2 (a) Write down an expression for the **exact** value of  $z$ , if  $\log_e z = 6$ .

(b) Find  $p$ , if  $p = \frac{\log_5 8}{\log_5 2}$ .

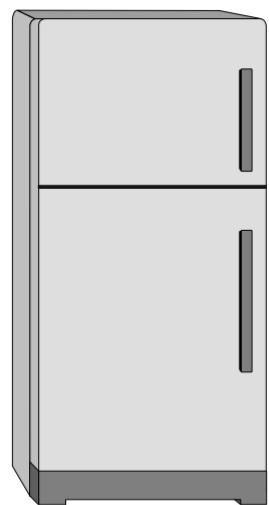
(c) Solve  $4^{x+1} = 3^{2-x}$ .

3 A developing country had a campaign to encourage people to purchase a refrigerator. The percentage,  $y$ , of households possessing refrigerators  $t$  years after the start of the campaign, is modelled by  $y = 100 - 95e^{-0.15t}$ .

(a) (i) Find the percentage of households that had refrigerators after 5 years.

(ii) Find the percentage of households that already had refrigerators at the start of the campaign.

(a) How many years will need to elapse before 90% of households have a refrigerator?



16. The diagram opposite shows the graph of  $y = \log_2 x$ .

(a) Find the value of  $a$ .

(b) Sketch the graph of  $y = \log_2 x - 3$

(c) Sketch the graph of  $y = \log_2 4x$

